

77. (a) Since the pressure (due to the water) increases linearly with depth, we use its average (multiplied by the dam area) to compute the force exerts on the face of the dam, its average being simply half the pressure value near the bottom (at depth  $d = 48$  m). The maximum static friction will be  $\mu N$  where the normal force  $N$  (exerted upward by the portion of the bedrock directly underneath the concrete) is equal to the weight  $mg$  of the dam. Since  $m = \rho_c V$  with  $\rho_c$  being the density of the concrete and  $V$  being the volume (thickness times width times height:  $\ell wh$ ), we write  $N = \rho_c \ell whg$ . Thus, the safety factor is

$$\frac{\mu \rho_c \ell whg}{\frac{1}{2} \rho_w g d A_{\text{face}}} = \frac{2 \mu \rho_c \ell wh}{\rho_w d (wd)} = \frac{2 \mu \rho_c \ell h}{\rho_w d^2}$$

which (since  $\rho_w = 1 \text{ g/cm}^3$ ) yields  $2(.47)(3.2)(24)(71)/48^2 = 2.2$ .

- (b) To compute the torque due to the water pressure, we will need to integrate Eq. 15-7 (multiplied by  $(d-y)$  and the dam width  $w$ ) as shown below. The countertorque due to the weight of the concrete is the weight multiplied by half the thickness  $\ell$ , since we take the center of mass of the dam is at its geometric center and the axis of rotation at  $A$ . Thus, the safety factor relative to rotation is

$$\frac{mg \frac{\ell}{2}}{\int_0^d \rho_w g y (d-y) w dy} = \frac{\rho_c \ell whg \frac{\ell}{2}}{\frac{1}{6} \rho_w g w d^3} = \frac{3 \rho_c \ell^2 h}{\rho_w d^3}$$

which yields  $3(3.2)(24)^2(71)/(48)^3 = 3.55$ .